Set

- A set is a collection of well defined object or things.

e.g. A = {1, 2, 3, 4}

Cardial number

n(A) = 4

Representation.

1) Rooster form
2) Set builder form

A = {a : a is a natural number till 4}

① Rooster Form

A Rooster Form is where elements are placed in braces {} and are seperated by commas.

A = {1, 2, 3, 4}

② Set builder form

- A set builder form it is a rule or statement describing common characteristics of all the elements written instead of writting elements directly in braces.

A = {a, i, e, o, u}

A = {a : a is a vowel}

Sub set

- A set 'A' is called subset of B if $x \in B \Rightarrow x \in A$

$A = \{a, i, e, o, u\}$

$B = \{a, i, e\}$

$A \subseteq B$

1st unit test

Types of sets

① Empty set - A set having no element is called as Empty set.
- It is denoted by $\emptyset$. $\{\}$

② Singletone set
- A set having only one element is called as singletone set.
- e.g. $A = \{2\}$

③ Finite set
- A set has a fix finite number of elements inside it is called as finite set.
- e.g.

$A = \{A\ a, b, c, d\}$

④ Infinite set
- A set has indefinite or infinite number of elements inside it is called as Infinite set.

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⑤ Equivalent

- If the number of elements present in two set are equal i.e. gardian number of two sets is same then this set is known as equivalent set.

⑥ Equal set

- If the number of elements and the also the element of two set are the same irrespective of the order then the set is Equal set.

⑦ Unequal set

- At least any one of the one set differs from elements of another set then the set is unequal set.

⑧ Universal set

- A universal set offen denoted by U.
- refers to the said that it contains all the element under consideration to the specific content or problem.

⑨ Disjoint set

- If Non of the element between two sets are common then they are called as Disjoint set.

Power set

- power set is a set x is a set of all possible subsets of set x including null set & x itself.

- If x has n elements then its power set will contain 2^n ~~ele~~ subsets.

e.g. $A = \{1, 2, 3\}$

$P(A) = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$.

Fundamental operation on sets.

1) Union

- A union of two or more set is a set that contains all the distinct elements of any of the given set.

e.g.

$A = \{a, b, c, d, e\}$

$B = \{d, f, g, a, z\}$

$A \cup B = \{a, b, c, d, e, f, g, z\}$

$= \{x \mid x \in A \text{ or } x \in B\}$

2) Intersection

– A Intersection of two or more sets is a set that contains all elements that are common to all the given sets.

e.g.

$A : \{1, 2, 3\}$

$B : \{2, 3, 5\}$

$A \cap B : \{2, 3\}$

3) Difference

– The difference betn two set offen refer as a "set diffrence, is a new set that contains all the elements belongs to first set but not second.

$A - B : \{b, c, e\}$

$A - B : \{x \mid x \in A \ \& \ x \notin B\}$

4) Cartesian product

– A cartesian product of two sets set p & set q denoted by $p \times q$

- Is a set of all possible ordered pairs (1, 3) where 1 represent element of p & 3 represents the element of q.

- In other words it combines each element from one set with another set.

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e.g. $p = \{1, 2\}$
$q = \{3, 4\}$

$p \times q = \{(1,3), (1,4), (2,3), (2,4)\}$

\# $A = \{3, 5, 6\}$
$B = \{9, 8, 1\}$
$C = \{7, 6, 11\}$

$A \times B = \{(3,9), (3,8), (3,1), (5,9), (5,8), (5,1), (6,9), (6,8), (6,1)\}$

$A \times C = \{(3,7), (3,6), (3,11), (5,7), (5,6), (5,11), (6,7), (6,6), (6,11)\}$

$A \times B \cap A \times C = \{\} = \phi$.

Venn diagram

$$n(A\cup B) = n(A) + n(B) - n(A\cap B)$$

Q. In a class of 40 students 18 likes mathematics 16 likes science & 10 likes both then find the students who like either math or science

→ Given

Let,

$n(A) = 18$ = Students who like Math

$n(B) = 16$ = Students who like science

$n(A\cap B) = 10$ = Students who like both.

$$n(A\cup B) = n(A) + n(B) - n(A\cap B)$$

$$= 18 + 16 - 10$$

$$n(A\cup B) = 24$$

Q. In a group of people 50 people either speak English or Hindi. 10 people prefer speaking both Hindi & English. 20 prefer English. How many ~~students~~ people prefer speaking Hindi.

→ $n(A\cup B) = 50$

$n(A\cap B) = 10$

$n(A) = 20$

$n(B) = ?$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$50 = 20 + n(B) - 10$

$50 + 10 = 20 + n(B)$

$n(B) = 40$.

1st unit test

Q. In a class students like to play this games Football, Cricket, Volly ball 5 students play all the three games 20 plays Football 80 plays Volly ball & 40 plays cricket 10 plays both Cricket & volly ball. 12 plays both football & cricket, 9 plays both Football & volly ball How many students are in class:

→

$n(F)$ = Student who plays Football
= 20

$n(C)$ = Students who plays cricket
= 40

$n(V)$ = Students who plays volly ball
= 80

$n(C \cap V) = 10$

$n(F \cap C) = 12$

$n(F \cap V) = 9$

$n(F \cup C \cup V) = ?$

$n(F \cap C \cap V) = 5$

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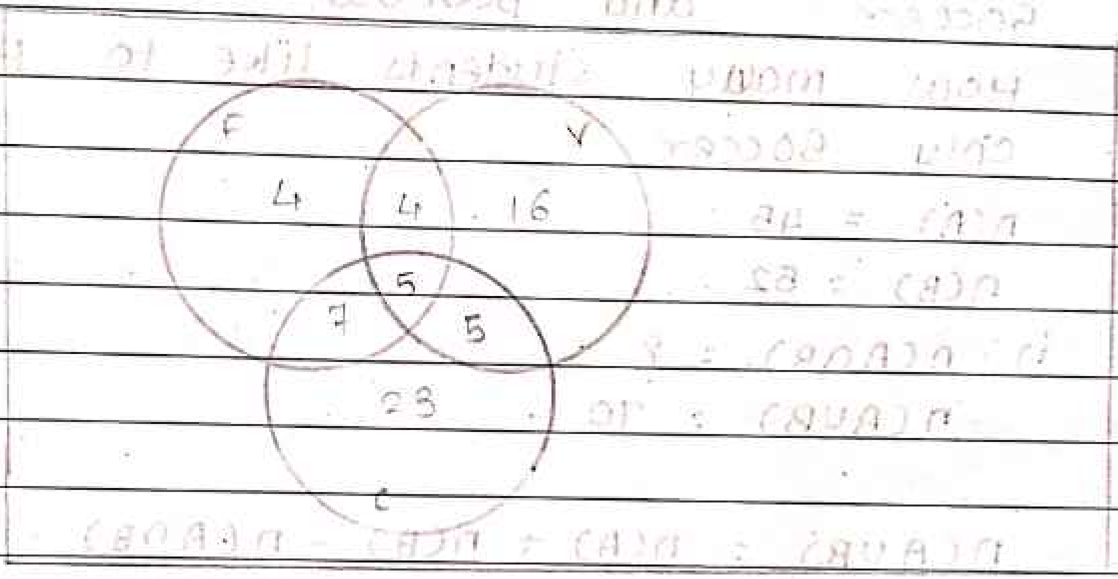
$$n(F \cup C \cup V) = n(F) + n(C) + n(V) - n(F \cap C) - n(C \cap V) - n(F \cap V) + n(F \cap C \cap V)$$

$$= 20 + 40 + 30 - 12 - 10 - 9 + 5$$

$$= 64$$



Consider a group of student in school let A be the set of student who play basketball & B be the set of students who play baseball. Given that the cardial number of A is 50 & cardial number of B is 40 & $n(A \cap B)$ is 15 find students who plays only basket ball & students who plays only baseball.

$n(A) = 50$

$n(B) = 40$

$n(A \cap B) = 15$

$n(A - B) = n(A) - n(A \cap B) = 35$

$n(B - A) = n(B) - n(A \cap B) =$

In a class of 70 students 45 plays soccer 52 likes to play baseball. All of the students likes to play one of the two games.

How many students likes to play soccer and baseball

How many students like to play only soccer

→ $n(A) = 45$

$n(B) = 52$

i) $n(A \cap B) = ?$

$n(A \cup B) = 70$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$70 = 45 + 52 - n(A \cap B)$

$n(A \cap B) = 45 + 52 - 70$

$= 97 - 70$

$n(A \cap B) = 27$

ii) $n(A - B) = ?$

$n(A - B) = n(A) - n(A \cap B)$

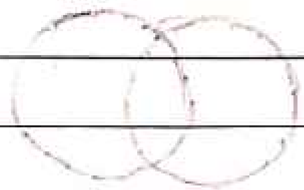
$= 45 - 27$

$= 18$

Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ using venn diagram

→

LHS



Propositional Logic & Logical Connectivities.

Logical connectivities	Symbol	Uses.
And / Conjuction	$\wedge$	$p \wedge q$
OR / Disjunction	$\vee$	$p \vee q$
Negation	$\sim$	$\sim p$
Conditional "if ... then ..."	$\Rightarrow$	$p \Rightarrow q$ / $p \rightarrow q$
Biconditional if & only if	$\Leftrightarrow$	$p \Leftrightarrow q$ $p \leftrightarrow q$
NAND (NOT + AND)	$\uparrow$	$p \uparrow q$
NOR (NOT + OR)	$\downarrow$	$p \downarrow q$

Conjunction

- Any two propositions can be combine by any word "AND" to form a compound proposition is said to be Conjuction .

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

eg.

Nashik is in India & $2+2=4$.

$p \wedge q \Rightarrow T \wedge T \Rightarrow T$

Dis

- Any two propositions can be combine by the word "OR" to form a compound is said to be Disjunction.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

e.g.

Nashik is in India & OR 2+2 = 4.

$p \vee q \Rightarrow T \vee T \Rightarrow T$

Negation.

- The Negation proposition of any given statement p is the proposition whose truth value is opposite to p.

p : The Flower is Pink

$\sim p$: The Flower is not pink

p	$\sim p$
T	F
F	T

- A proposition is said to be totology if it only contains T in last coloumn of the truth table & If It contain only F in last coloum of Truth table then it is contradiction

Q. $\{(P \vee \sim Q) \wedge (\sim P \vee \sim Q) \vee Q\}$

I	II	III	IV	V	VI
p	q	~p	~Q	P ∨ ~Q	~P ∨ ~Q
T	T	F	F	T	F
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	T	T

VII	VIII
V ∧ VI	VII ∨ II
(P∨~Q) ∧ (~P ∨~Q)	(P∨~Q) ∧ (~P∨~Q) ∨ Q
F	T
T	T
F	T
T	T

∴ Given statement is Tautology

$P \vee \sim(q \wedge r)$ & $(P \vee \sim q) \vee \sim r$

1	2	3	4	5	6	7
P	q	r	~q	~r	(q∧r)	~(q∧r)
T	T	T	F	F	T	F
T	T	F	F	T	F	T
T	F	T	T	F	F	T
T	F	F	T	T	F	T
F	T	T	F	F	T	F
F	T	F	F	T	F	T
F	F	T	T	F	F	T
F	F	F	T	T	F	T

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8	9	10	11
1 ∨ 7	$(p \vee \sim q)$	9 ∨ 5	8 ∧ 10
T	T	T	T
T	T	T	T
T	T	T	T
T	T	T	T
F	F	F	F
T	F	T	T
T	T	T	T
T	T	T	T

Q. $(p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$

→

1	2	3	4	5	6	7
p	q	$\sim p$	$\sim q$	$p \vee q$	$(p \vee \sim q)$	5 ∧ 6
T	T	F	F	T	T	T
T	F	F	T	T	T	T
F	T	T	F	T	F	F
F	F	T	T	F	T	F

8	9	10	11
$(\sim p \vee q)$	$(\sim p \vee \sim q)$	8 ∧ 9	7 ∧ 10
T	F	F	F
F	T	F	F
T	T	T	F
T	T	T	F

Q. $Q \vee (p \vee \sim q) \vee (\sim p \vee \sim q)$

p	q	$\sim p$	$\sim q$	$(p \vee \sim q)$	$Q \vee (p \vee \sim q)$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	T	T

$\sim p \vee \sim q$	$Q \vee (p \vee \sim q) \vee (\sim p \vee \sim q)$
F	T
T	T
T	T
T	T

$\therefore$ Given statement is Tautology.

Conditional statement

- Many statement are of the form if p then q such statement are said to be conditional statement

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

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Q. $(p \wedge q) \rightarrow (p \vee q)$

→

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Q. $p \rightarrow (p \wedge (q \rightarrow p))$

→

p	q	$q \rightarrow p$	$p \wedge (q \rightarrow p)$	$p \rightarrow (p \wedge (q \rightarrow p))$
T	T	T	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

~~Q.~~ ~~(p∨q~~

Biconditional statement

- A statement p "if & only if" q

Such statement are called Biconditional statement

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

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Q $(p \vee q \vee r) \leftrightarrow \{[((p \rightarrow q) \rightarrow q) \rightarrow r] \rightarrow r\}$

r

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$p \rightarrow q$	$(p \rightarrow q) \rightarrow q$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	F
F	F	F	F	F	T	F

A $((p \rightarrow q) \rightarrow q) \rightarrow r$	x $A \rightarrow r$	$\cancel{x}\ r \leftrightarrow x$
T	T	T
F	T	T
T	T	T
F	T	T
T	T	T
F	T	T
T	T	T
T	F	T

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Q → $p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

$p \wedge q$	$(p \wedge q) \rightarrow r$
T	T
T	F
F	T
F	T
F	T
F	T
F	T
F	T

$\therefore \; p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

Methods of proof

Disjunctive normal form (DNF)

- DNF express a logical formula as a series of or operation (v) with in parenthesis, where each grouping represents the condition under which the output

+) This form is

e.g.

2) conjuctive

- CNF is opposite of DNF
- It expresses a logical formula an series of logical operation within parenthesis where each group represents a certain condition under which the output is false.

e.g.

- $(P \wedge q)$
- $(\sim p \vee q) \wedge (\sim p \vee r)$

Idempotent law

- $p \wedge p \leftrightarrow p$ $\quad$ $\sim p \wedge \sim p \leftrightarrow \sim p$
- $p \vee p \leftrightarrow p$ $\quad$ $\sim p \vee \sim p \leftrightarrow \sim p$

commulative law

$p \wedge q \leftrightarrow q \wedge p$

$p \vee q \leftrightarrow q \vee p$

Associative law

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

De-Morgan law

$\sim (p \wedge q) \leftrightarrow \sim p \vee \sim q$

$\sim (p \vee q) \leftrightarrow \sim p \wedge \sim q$

Distributive law

$p \wedge (q \wedge r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \leftrightarrow (p \vee q) \wedge (p \vee r)$

Some common laws

- $\cancel{p} \leftrightarrow$ $\quad p \rightarrow q \leftrightarrow \sim p \vee q$

$p \vee \sim p \leftrightarrow 1$

$\sim p \wedge p \leftrightarrow 1$

$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

$(\sim p \leftrightarrow q) \leftrightarrow p \vee q$

DNF

1) $(p \rightarrow q) \wedge (\sim p \wedge q)$

$(\sim p \vee q) \wedge (\sim p \wedge q)$

$(\sim p \vee q) \wedge A$

$(\sim p \wedge A) \vee (q \wedge A)$

$(\sim p \wedge \sim p \wedge q) \vee (q \wedge \sim p \wedge q)$

$(\sim p \wedge q) \vee (q \wedge \sim p)$

CNF

$(p \wedge q) \vee (\sim p \wedge q \wedge r)$

Obtain DNF OF

$p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r)))$

$\rightarrow p \vee (\sim p \rightarrow (q \vee (\sim q \vee \sim r)$

$p \vee (\sim p \rightarrow (q \vee \sim q) \vee (q \vee \sim r)$

$p \vee (\sim p \rightarrow (q \vee \sim r))$

$p \vee (\sim p \rightarrow (q \vee \sim r))$

$p \wedge (p \vee (q \vee \sim r)$

$p \vee ((p \vee q) \vee (p \vee \sim r))$

$p \vee [p \vee q \vee p \vee \sim r]$

$p \vee (p \vee q \vee \sim r)$

$(p \vee p) \vee (p \vee q) \vee (p \vee \sim r)$

$p \vee (p \vee q) \vee (p \vee \sim r)$

$p \vee p \vee q \vee p \vee \sim r$

$p \vee q \vee \sim r$

Obtain CNF OF

$(p \rightarrow q) \wedge (q \vee (p \wedge r))$

$\rightarrow (p \rightarrow q) \wedge (q \vee (p \wedge r))$

$(\sim p \vee q) \wedge (q \vee (p \wedge r))$

$(\sim p \vee q) \wedge (q \vee p) \wedge (q \vee r)$

Truth table

				A	B	C
p	q	r	~p	~p∨q	q∨p	q∨r
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	F	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	T	F	T
F	F	F	T	T	F	F

D	
A∧B	D∧C
T	T
T	T
F	F
F	F
T	T
T	T
F	F
F	F

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Predicate

- A predicate is a sentence that contains a finite numbers of variables and becomes a proposition when a specific values are substituted for the variables where $P(x)$ is a propositional function or a predicate and x is a predicate variable.

Domain

- Domain of the predicate variable is the set of all possible values that may be sustituted in a place of variable

Universal quantifier ($\forall$)

- The universal Quantifier ($\forall$) is used to make a statement that applies every element in set. It is offen represent that for All for Every

Existantial quantifier ($\exists$)

- The existantial quantifier denoted by $\exists$
- Is used to make a statement that asserts the existance of at least one element in a set that satisfy a perticular condition

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- It is often refered as it is exist

Q Let $D = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
Determine the truth value

1) $(\forall x \in D),\ x+4 < 15$
→ True. For all values of x, $x+4 < 15$

2) $(\exists x \in D),\ x+4 = 10$
→ True. there exists $x = 6$

3) $(\forall x \in D),\ x+4 \le 10$
→ False

4) $(\exists x \in D),\ x+4 > 15$
→ False

#

- It is a second method of proof technique used in mathematics to establish the truth of a statement for all integers greater than or equal to some starting point, typically denoted as 'n'.
- It consists of two steps.
  ① Base step
  - proof that the statement is

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wholes true for the smallest value n usually zero or one depending on the context.

② Inductive step
- Assume that the statement is true for an arbitary ~~Posive~~ positive integer "k" then using these assumption proof that it must also be true for k+1

unit

Q. Show that in M.I $n \geq 1$ for

$$1+2+3+4+5+\cdots\cdots+n = \frac{n(n+1)}{2}$$

→ Let.

$$P(n) = 1+2+3+4+\ldots+n = \frac{n(n+1)}{2}$$

① Base step.

assume $n=1$

$$P(1) = \frac{1(1+1)}{2}$$

$$1 = 1$$

P(1) is true

② Induction step

We must prove that $k \geq 1$

P(k) is true.

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$$1+2+3+\cdots+k = \frac{k(k+1)}{2} \quad \text{--- (1)}$$

also show $p(k+1)$ is true

$$1+2+3+\cdots+k+(k+1) = \frac{(k+1)((k+1)+1)}{2}$$

$$\frac{k(k+1)}{2} + \frac{(k+1)}{\cancel{2}} = \frac{(k+1)(k+2)}{2}$$

$$\frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$

$$\frac{(k+1)(k+2)}{2} = \frac{(k+1)(k+2)}{2}$$

LHS = RHS.

Q. Show that in M.I $n \geq 1$. for

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

→ Let $p(n)$: $1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

(1) Base step

assume $n=1$

$$P(1) = \frac{1(1+1)(2+1)}{6}$$

$$1 = \frac{(2)(3)}{6}$$

$$1 = \frac{6}{6}$$

$$1 = 1$$

P(1) is true

② Induction step

We must prove that $k \geq 1$, $P(k)$ is true

$$1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6} \quad \text{— ①}$$

We must prove for $P(k+1)$

$$1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

$$\frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$LHS = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$= \frac{(k+1)}{6}\left[k(2k+1) + 6(k+1)\right]$$

$$= \frac{k+1}{6}\left[2k^2 + k + 6k + 6\right]$$

$$= \frac{k+1}{6}\left[2k^2 + 7k + 6\right]$$

7

12

4 3

$$= \frac{k+1}{6}\left[2k^2 + 4k + 3k + 6\right]$$

$$= \frac{k+1}{6}\left[2k[k+2] + 3[k+2]\right]$$

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$$= \frac{k+1}{6}\left[(2k+3)(k+2)\right]$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$\therefore$ LHS = RHS.

Q. Show that in M.I $n \geq 1$ for

$$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$$

→ Let.

$$P(n) : 1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$$

① Base step.

$$P(n) = \frac{n(2n+1)(2n-1)}{3}$$

$$P(1) = \frac{1(2+1)(2-1)}{3}$$

$$= \frac{3 \times 1}{3}$$

$$1 = 1$$

P(1) is true.

② Induction step.

We must prove that $k \geq 1$

$$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$$

We have to prove P(k+1)

$$= \frac{k+1}{6} \left[(2k+3)(k+2) \right]$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$\therefore$ LHS = RHS.

Q. Show that in M.I $n \geq 1$ for

$$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$$

→ let,

$$P(n) : 1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$$

① Base step.

$$P(n) = \frac{n(2n+1)(2n-1)}{3}$$

$$P(1) = \frac{1(2+1)(2-1)}{3}$$

$$= \frac{3 \times 1}{3}$$

$$1 = 1$$

P(1) is true.

② Induction step.

We must prove that $k \geq 1$

$$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$$

We have to prove $P(k+1)$

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$$1^2+3^2+5^2+\dots+\underbrace{(2(k+1)-1)^2}_{(2k-1)^2} = \frac{(k+1)(2(k+1)+1)(2(k+1)-1)}{3}$$

$$1^3+3^2+5^2+\dots+\underbrace{(2k+2-1)^2}_{(2k-1)^2} = \frac{(k+1)(2k+2+1)(2k+2-1)}{3}$$

$$1^3+3^2+5^2+\dots+\underbrace{(2k+1)^2}_{(2k-1)^2} = \frac{(k+1)(2k+3)(2k+1)}{3}$$

$$\frac{k(2k+1)(2k-1)}{3} + (2k+1)^2 = \frac{(k+1)(2k+3)(2k+1)}{3}$$

$$\frac{k(2k+1)(2k-1)+3(2k+1)^2}{3} = \frac{(k+1)(2k+3)(2k+1)}{3}$$

$$LHS = \frac{k(2k+1)(2k-1)+3(2k+1)^2}{3}$$

$$= \frac{k(2k+1)(2k-1)+3(4k^2+4k+1)}{3}$$

$$= \frac{k([2k]^2-(1)^2)+3(4k^2+4k+1)}{3}$$

$$= \frac{4k^3-k^2+3(4k^2+4k+1)}{3}$$

$$= \frac{k^2(4k-1)+3(4k^2+4k+1)}{3}$$

$$= \frac{4k^3-k^2+12k^2+12k+3}{3}$$

$$\frac{(2k+1)}{3}\left[k(2k-1)+3(2k+1)\right]$$

$$=\frac{2k+1}{3}\left[2k^2-k+\cancel{3}6k+3\right]$$

$$=\frac{2k+1}{3}\left[2k^2+5k+3\right]$$

$$=\cancel{\frac{2k+1}{3}}\left[\cancel{2k^2+6k-1k+}\right.$$

$$=\frac{2k+1}{3}\left[2k^2+2k+3k+3\right]$$

$$=\frac{2k+1}{3}\left[2k(k+1)+3(k+1)\right]$$

$$=\frac{2k+1}{3}\left[(k+1)(2k+3)\right]$$

$$=\frac{(2k+1)(k+1)(2k+3)}{3}$$

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By using principle of M I . prove that $n(n^2+5)$ is an integer multiple of 6 for all positive integer

→ let $p(n)$

$6n = n(n^2+5)$

① Basic step

Let $n=1$

$P(1) = 1(1^2+5)$

$6 = 1(6)$

$6 = 6$

$P(1)$ is true.

② Induction step .

We must show for $k \geq 1$, $P(k)$ is true

$6k = k(k^2+5)$ —①

Now,

show for $p(k+1)$

$P(k+1) = (k+1)((k+1)^2+5)$

$= (k+1)(k^2+2k+1+5)$

$= k^3+2k^2+k+5k+k^2+2k+1+5$

$= k^3+3k^2+3k+5k+6$

$= k^3+5k+3k^2+3k+6$

$= k(k^2+5)+3k^2+3k+6$

$= 6k+3(k^2+k+2)$

$P(k+1)$ is true

GCD

Greatest common Division.

– Euclidean Algorithm

① GCD OF (12, 33)

Dividend Division

Q	A	B	R
2	33	12	9
1	12	9	3
3	9	3	0
	3	0	

② GCD (750, 900)

Q	A	B	R
1	900	750	15
30	750	15	0
	15	0	

③ GCD (252, 105)

Q	A	B	R
2	252	105	42
2	105	42	21
2	42	21	0
	21	0	

④ GCD (1005 . 105)

→

Q	A	B	R
9	1005	105	60
1	105	60	45
1	60	45	15
3	45	15	0
	15	0	

⑤ GCD (83 . 19)

Q	A	B	R
4	83	19	7
2	19	7	5
1	7	5	2
2	5	2	1
2	2	1	0
	1	0	

⑥ (529 . 123)

Q	A	B	R
4	529	123	31
3	123	31	25
1	31	25	6
4	25	6	1
6	6	1	0
	1	0	

#

If p belongs to ($p \in Z^+$) positive integer and p is greater than 1 is consider prime if the only factor of p are 1 & p otherwise the number is composite this means the number is composit iff $\exists\ a \in Z^+$. Such that $a \mid p$ and 1 is less than a is ~~is~~ less than p ($1 < a < p$)

* Fundamental theorem of Arithmatic.

Every positive integer greater than 1 can be written uniquely as a prime of the product of 2 where primes are written in non-decending order.

Among the integer 1 to 1000 :

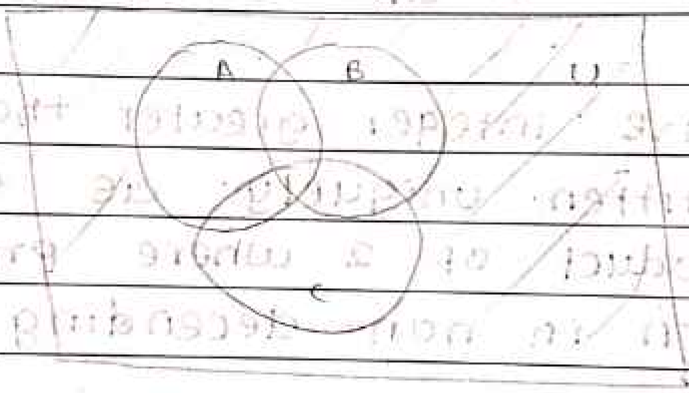
i) How many are not divisible by 3 nor by 5 nor by 7

A : Divisible by 3

B : Divisible by 5

C : Divisible by 7

$(A \cup B \cup C)' = U - n(A \cup B \cup C)$



$n(A) = \frac{1000}{3} \approx 333$

$n(B) = \frac{1000}{5} \approx 200$

$n(C) = \frac{1000}{7} = 142$

$n(A \cap B) = 3 \times 5 = \frac{1000}{15} \approx 66$

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$n(A \cap C) = 3 \times 7 = \frac{1000}{21} = 48$

$n(B \cap C) = 5 \times 7 = \frac{1000}{35} = 29$

$n(A \cap B \cap C) = \frac{1000}{3 \times 5 \times 7}$

≈ 10

$n(A \cup B \cup C) = n(A) + n(B) + n(C) -$

$n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$= 333 + 200 + 142 - 66 - 48 - 29 + 10$

$= 542$

$(A \cup B \cup C)' = U - n(A \cup B \cup C)$

$= 1000 - 542$

$= 458$

$n(A) = 333$

$n(B) = 200$

$n(C) = 142$

Unit - II

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Function

- Function

  A Function (F) from A to B is an assignment of exactly one element of B to each element of A where A & B are non-empty set.

  Here A is called domain and B is called co-domain

- Domain and co-domain

  - If f is a function from A to b then a is called domain & B is called co-domain

- Range

  Range of f is a set of all image of element of A.

  r is subset of B

- Injection (one to one function)

  Injective

  - A function is called one to one function if for all element a & b in A

$$f(a) = f(b)$$

$$a = b$$

$$\forall a \, \forall b \, (f(a) = f(b) \rightarrow a = b)$$

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OR

$\forall a \ \forall b \ (f(a) \neq f(b) \longrightarrow a \neq b)$

$f(x) = x^2$

A	B
-2	-4
	-3
-1	-2
	-1
0	0
1	+1
	2
2	3
	4

$(-2) \neq (2)$

$f(-2) = f(2)$

$f(x) = f(y)$

$x^2 = y^2$

$x^2 - y^2 = 0$

$(x+y)(x-y) = 0$

$x + y = 0 \qquad x - y = 0$

$x = -y \qquad x = y$

$\therefore$ It is not one to one function

Surjective Function
(on to Function)

- If every element b in a set B
A corresponding element a in a set A
Such that

$$F(a) = b$$

Such function is called
Surjective function

The range of surjective function
is same as co-domain

Bijective Function

A Function is Bijective if it
contain both one to one and
Onto Function

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Q. $F(x) = 2x + 3$

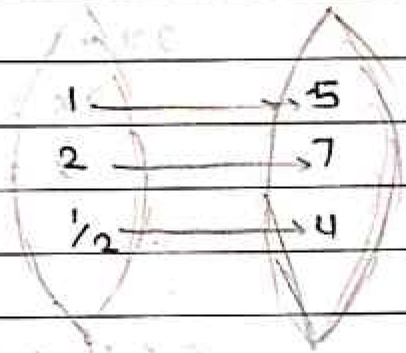
Domain - $\{1, 2, 1/2\}$

Co-domain - $\{5, 7, 4\}$

$F(1) = 2(1) + 3$

$= 2 + 3$

$F(1) = 5$



$f(2) = 2 \times 2 + 3$

$= 4 + 3$

$= 7$

$f(1/2) = 2 \times 1/2 + 3$

$= 1 + 3$

$= 4$

Q. $F(x) = 3x - 2$ Determine the type of function

→

① One to one function

$f(x) = f(y)$

$3x - 2 = 3y - 2$

$3x = 3y$

$x = y$

∴ It is one to one function.

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Q. $f(x) = 2x+1$ prove that it is on to function.

→

On to function

$$f(x) = y$$

$$2x+1 = y$$

$$2x = y-1$$

$$x = \frac{y-1}{2}$$

$$f\left(\frac{y-1}{2}\right) = 2\left(\frac{y-1}{2}\right)+1$$

$$= y-1+1$$

$$= y$$

Inverse of a Function

Let f be the function from A to B with a Bijection then $f^{-1}: B \to A$ which associates each element b belongs to set B to a diffrent element a belongs to set A. such that $f(a) = b$ then $f^{-1}(b) = a$

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Relation.

A Relation or binary relation are from set A to set B is a subset of $A \times B$ which can be defines as $aRb \mid R(a, b)$.

aRb

$R(a, b)$

$A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(a, b) \mid a \text{ divides } b\}$

$a = \{1, 2, 3, 4, 5, 6\}$

$b = \{1, 2, 3, 4, 5, 6\}$

$A \times A = \{(1,1), (1,2), (1,3), (1,4), (1,5),$
$(1,6), \ldots \ldots (6,6)\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (1,5),$
$(1,6), (2,2), (2,4), (2,6), (3,3),$
$(3,6), (4,4), (5,5), (6,6)\}$

$R \subseteq A \times A$

$a R b$

$R(a,b)$

$A = \{1, 2, 3, 4\}$

$B = \{a, b, c\}$

$R = \{(1,a), (1,b), (2,c), (3,a)\}$

$\overline{R} = \{(a,b) \mid (a,b) \notin R\}$

$\overline{R} = \{(1,c), (2,a), (2,b), (3,b), (3,c)$
$(4,a), (4,b), (4,c)\}$

$R^c = \{(b,a) \mid (a,b) \in R\}$

$R^c = \{(a,1), (b,1), (c,2), (a,3)\}$

$a R b$

$R(a,b)$

$A = \{2, 4, 6, 3\}$

$R = \{(a,b) \mid a = b+1 \text{ or } b = 2a\}$

$a = \{2, 4, 6, 3\}$ $b = \{2, 4, 6, 3\}$

$R = \{(3,2), (4,3), (2,4), (3,6)\}$

$$M_R = \begin{array}{c} \\ 2 \\ 4 \\ 6 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 2 & 4 & 6 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

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Types of Relation.

1) Reflexive Relation.

- A relation are on a small set A is called a reflexive relation if $\forall a\ ((a,a) \in R)$

2) Symmetric Relation.

- A relation R on a set A is called as a Symmetric relation if -
$\forall a\ \forall b\ ((a,b) \in R \rightarrow (b,a) \in R)$
For ex -
$(1,2) \longrightarrow (2,1)$

Q. $A = \{2, 4, 6, 8\}$
$R = \{(a,b) \mid a \text{ divide } b\}$

$R = \{(2,4), (2,6), (2,8), (4,8), (4,4)$
$(2,2), (6,6), (8,8)\}$

$\therefore$ This is reflecxive relation

M^R :

	2	4	6	8
2	1	1	1	1
4	0	1	0	1
6	0	0	1	0
8	0	0	0	1

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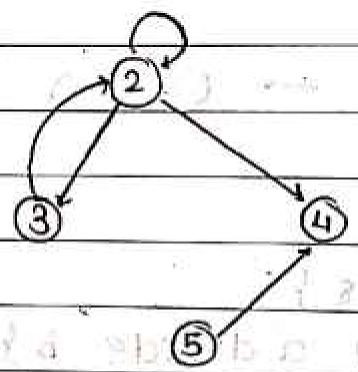
3) Transitive Relation

- A relation R is called as transitive relation if
$\forall a \forall b \forall c \, (((a,b) \in R \wedge (b,c) \in R) \rightarrow (a,c) \in R)$

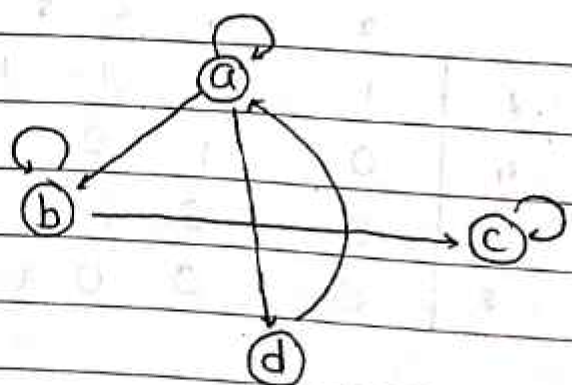
Graphical

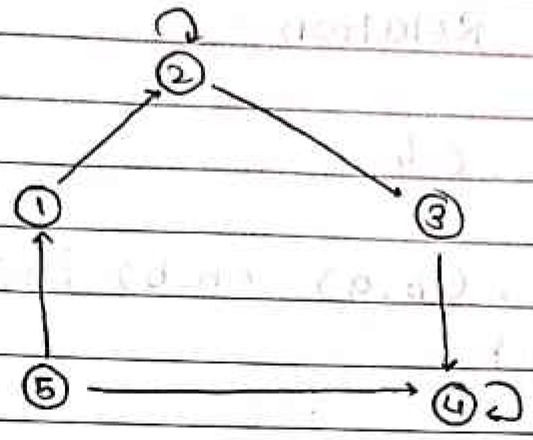
$A = \{2, 3, 4, 5\}$

aRb : $R = \{(2,3), (2,2), (2,4), (3,2), (5,4)\}$



$R = \{(a,a), (a,b), (a,d), (b,b), (b,c), (c,c)$
$(c,d), (d,a)\}$





$R = \{(2,2), (4,4), (1,2), (5,1), (5,4), (3,4), (2,3)\}$

Q. $A = \{1, 2, 3, 4\}$

$B = \{1, 4, 6, 8, 9\}$

aRb iff $b = a^2$

→

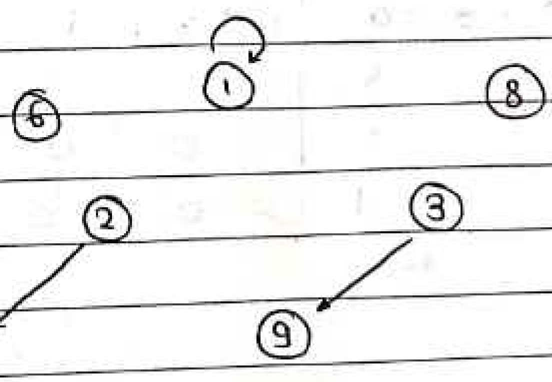
$R = \{(2,4), (1,1), (3,9)\}$

Domain $= \{2, 1, 3\}$

Range $= \{4, 1, 9\}$

$$M_R = \begin{array}{c|ccccc} & 1 & 4 & 6 & 8 & 9 \\ \hline 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Digraph -



Equivalence Relation

$A = \{a, b, c\}$

$R = \{(a,a), (b,a), (b,b), (c,c), (d,d), (a,b), (c,d), (d,c)\}$

① Reflexive - $(a,a), (b,b), (c,c), (d,d)$

② symmetric -

$(b,a) \rightarrow (a,b)$, $(c,d) \rightarrow (d,c)$

③ Transitive -

$(a,a) \wedge (a,b) \rightarrow (a,b)$

$(b,a) \wedge (a,b) \rightarrow (b,b)$

$(b,b) \wedge (b,a) \rightarrow (b,a)$

$(c,c) \wedge (c,d) \rightarrow (c,d)$

$(a,b) \wedge (b,a) \rightarrow (a,a)$

$(a,b) \wedge (b,b) \rightarrow (a,b)$

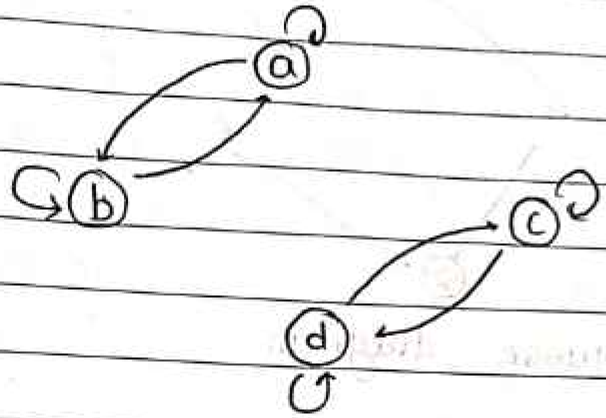
$(c,d) \wedge (d,d) \rightarrow (c,d)$

$(c,d) \wedge (d,c) \rightarrow (c,c)$

$(d,d) \wedge (d,c) \rightarrow (d,c)$

$$M^R = \begin{array}{c|cccc|} & a & b & c & d \\ \hline a & 1 & 1 & 0 & 1 \\ b & 1 & 1 & 0 & 1 \\ c & 0 & 0 & 1 & 1 \\ d & 0 & 0 & 1 & 1 \end{array}$$

Digraph.



Partial order Relation

:- A Binary relation are on a non empty set A is called the partial order if R is reflexive, antisymmetric & transistive

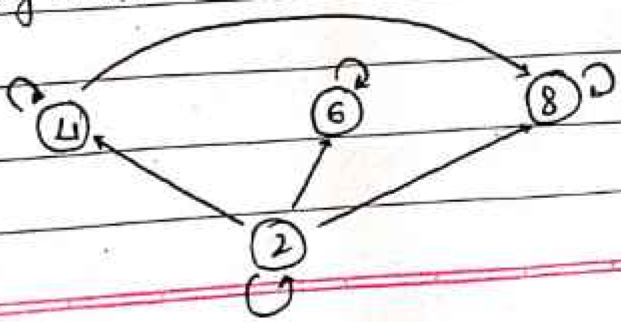
Ex -

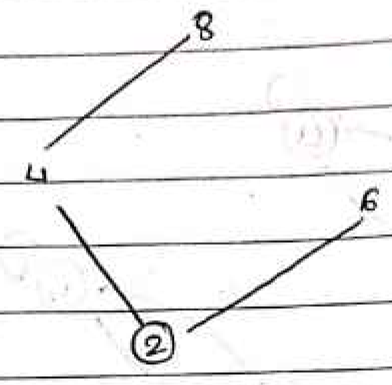
A = {2, 4, 6, 8}
R iff a divides b

→ aRb : R = { (2,4), (2,6), (2,8), (4,8), (2,2), (4,4), (6,6), (8,8) }

Hass diagram.

diagraph





Hasse diagram.

Q. $m = 6$, $m = 45$, $m = 12$.

$A = \{1, 2, 3, 6\}$

$R = \{(1,1), (1,2), (1,3), (1,6), (2,2), (2,6), (3,3), (3,6), (6,6)\}$

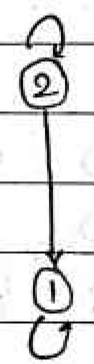
$R = \{(x, y) \mid x \in A \wedge y \in A \wedge y/x\}$

1) $m = 2$

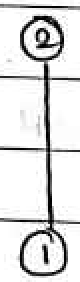
$A = \{1, 2\}$

$R = \{(1,1), (1,2), (2,2)\}$

Diagraph



Hasse Diagraph



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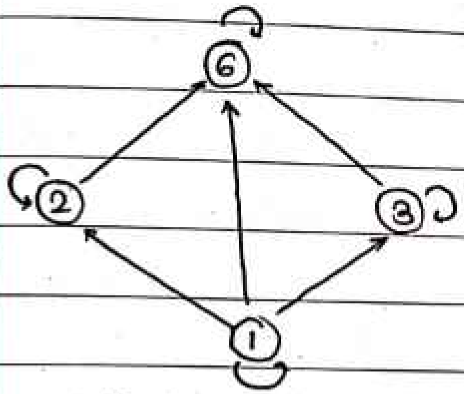
2) m : 6

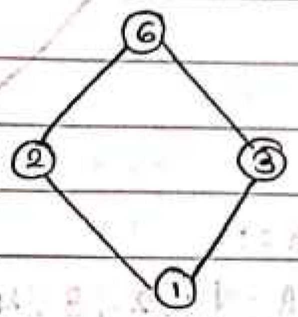
A = { 1, 2, 3, 6 }

R : { (1,1), (1,2), (1,3), (1,6), (2,2), (2,6), (3,3), (3,6), (6,6) }

Diagraph



Hasse diagram

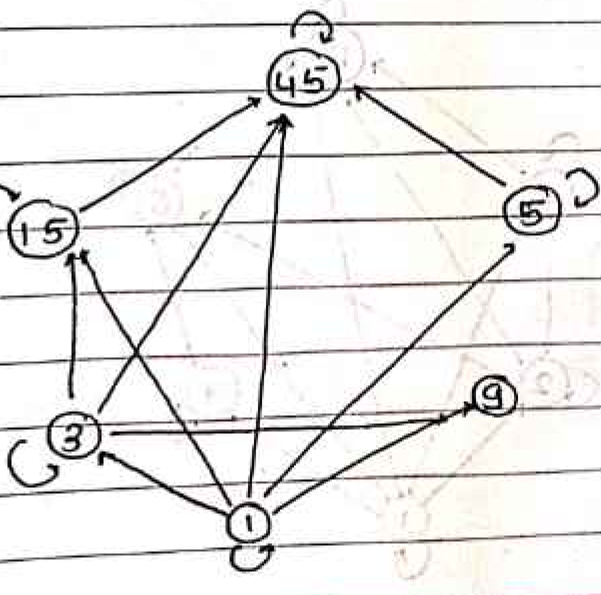


3) m : 45

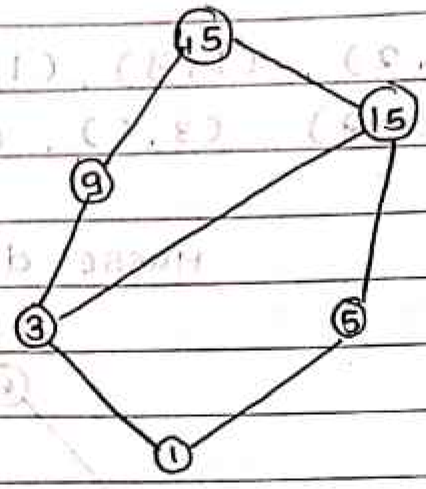
A : { 1, 3, 5, 9, 15, 45 }

R : { (1,1), (1,3), (1,5), (1,9), (1,15), (1,45), (3,3), (3,9), (3,15), (3,45), (5,5), (5,15), (5,45), (9,45), (9,9), (45,45), (15,15), (15,45) }

Diagraph



Hasse Diagram.

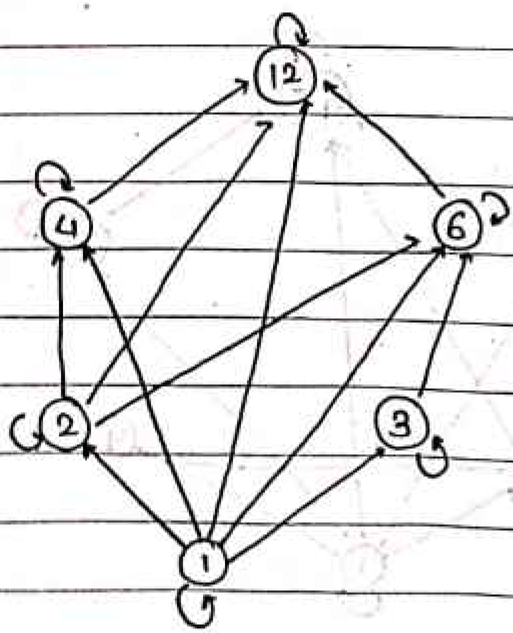


m = 12

A = {1, 2, 3, 4, 6, 12}

R = { (1,1), (1,2), (1,3), (1,4), (1,6), (1,12),
(2,2), (2,4), (2,6), (2,12),
(3,3), (3,6), (3,12)
(4,4), (4,12)
(6,6), (6,12)
(12,12) }

Diagraph.



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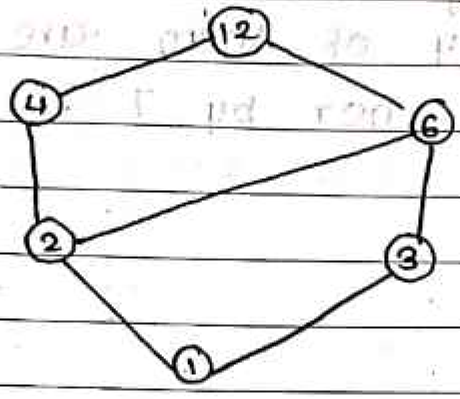
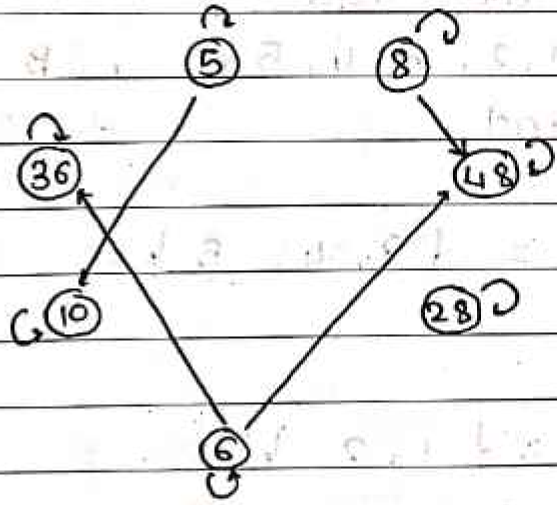
Hasse diagram



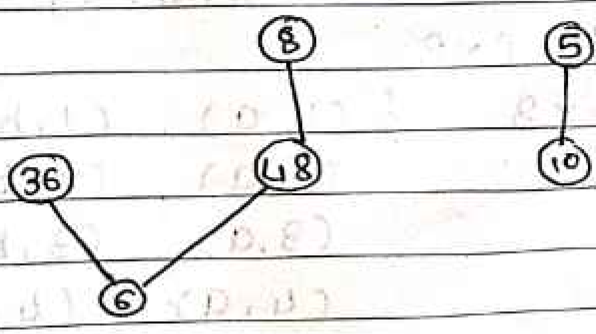
$A = \{6, 10, 28, 36, 48, 5, 8\}$

$R = \{(6,6), (6,36), (6,48), (10,10),$
$(5,5), (5,10), (8,8), (8,48),$
$(28,28), (36,36), (48,48)\}$

Diagraph



Hasse diagram



Q.B.

① Among integers 1 to 1000.
i) How many of them are not divisible 3 nor by 5 nor by 7 ?

→

Q. Given two sets.
A = {1, 2, 3, 4, 5} , B = {3, 4, 5, 6, 7}
Then find.
① A∪B
→ A∪B = {3, 4, 5}

② A - B
→ A - B = {1, 2}

Q. If A = {1, 2, 3, 4, 5} , B = {a, b, c}
Find A×B
→ A×B = {(1, a), (1, b), (1, c),
(2, a), (2, b), (2, c),
(3, a), (3, b), (3, c),
(4, a), (4, b), (4, c)
(5, a), (5, b), (5, c)}

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Let p & q be the proposition

p : Swimming at New Jersey shore is allowed.

q : Sharks have been spotted near the shore

i) $\sim q$

→ Sharks have been not spotted near shore.

ii) $\sim p$

→ Swimming at New Jersey shore is not allowed.

iii) $p \rightarrow \sim q$

→ If Swimming at New Jersey shore is allowed then Sharks have been not spotted near shore.

iv) $\sim p \vee q$

→ Swimming at New Jersey shore is not allowed or Sharks have been spotted near the shore

Explain universal quantifier & Existential quantifier with example. What is De Morgans law for Quantifiers?

→

① Universal Quantifier ($\forall$)

— The symbol $\forall$ (For all) represent a universal quantifier.

— It is used to make a statement that applies to every element in a given set or domain.

For e.g.

- $\forall x \; P(x)$ means for all x, $P(x)$ is true.
- $A = \{1, 2, 3, 4\}$
  $(\forall x \in A), \; x + 1 < 6$
  This means for all x $P(A)$ is true.

② Existential Quantifier ($\exists$)

- The symbol $\exists$ (there exists) represents an Existential Quantifier.
- It is used to make a statement that asserts the existance of at least one element in a set for which a given condition is true

For e.g.

$\exists x\, Q(x)$ means there exists an x such that $Q(x)$ is true

$A = \{1, 2, 3, 4, 5\}$

$(\exists x \in A),\ x + 2 = 5$

$\therefore$ True, there exists $x = 3$.

Demorgon's Law for Quantifier.

$$\neg \forall x \in U\, P(x) \leftrightarrow \exists x \in U\, (\neg P(x))$$

Q. Given $A = \{1, 2, 3, 4\}$ & $B = \{x, y, z\}$ let R be the following relation from A to B.

$R = \{(1, y), (1, z), (3, y), (4, x), (4, z)\}$

i) Determine the matrix of relation

$\rightarrow$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

ii) Find converse of Relation

$\rightarrow$ $R^{-1} = \{(y, 1), (z, 1), (y, 3), (x, 4), (z, 4)\}$

Determine whether the following proposition is contradicton or tautology

$(p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$

→

p	q	$\sim p$	$\sim q$	$p \vee q$	$p \vee \sim q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	F	T	T	F	T

A $(p \vee q) \wedge (p \vee \sim q)$	$(\sim p \vee q)$	$(\sim p \vee \sim q)$
T	T	F
T	F	T
F	T	T
F	T	T

B $(\sim p \vee q) \wedge (\sim p \vee \sim q)$	$A \wedge B$
F	F
F	F
T	F
T	F

The proposition is contradiction

$p \vee \bar{q} \vee \bar{p}$
$p \vee \bar{q}$

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Draw truth table for

$p \vee \sim q \vee \sim p$

p	q	$\sim p$	$\sim q$	$p \vee \sim q$ (A)	$A \vee \sim p$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	T	T

$p \vee \sim q$

T   T V T = T

T   T V F = T

F   F V T = T

T   T V F = F

Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} is R = {(a, b) : b = a+1} is reflexive, symmetric or transitive. Justify your answer. Find relation Matrix.

1+1=2
b=a+1

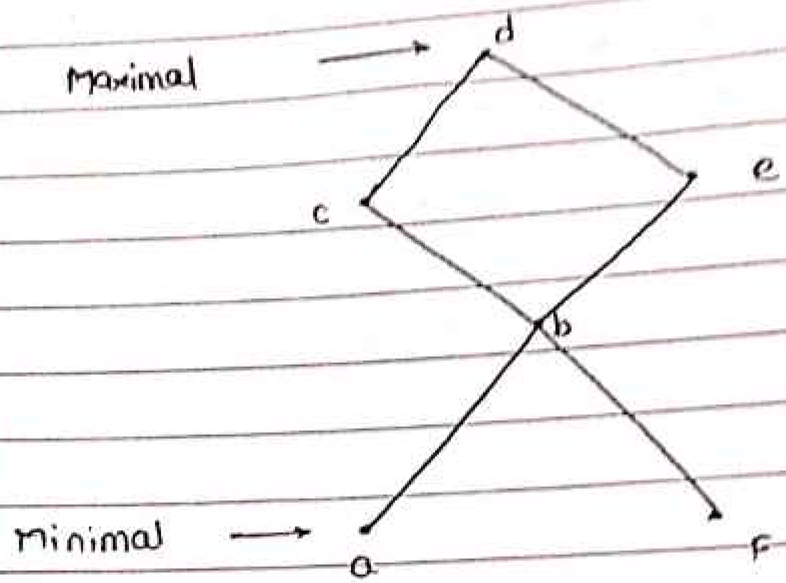
R = {(a, b) : b = a+1}   3 = 2   (1, 2) a b
(2, 3)

R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

$(a, b) \wedge (b, c) \rightarrow (a, c)$

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Lattice



upper bound {a, f} → b, c, e, d

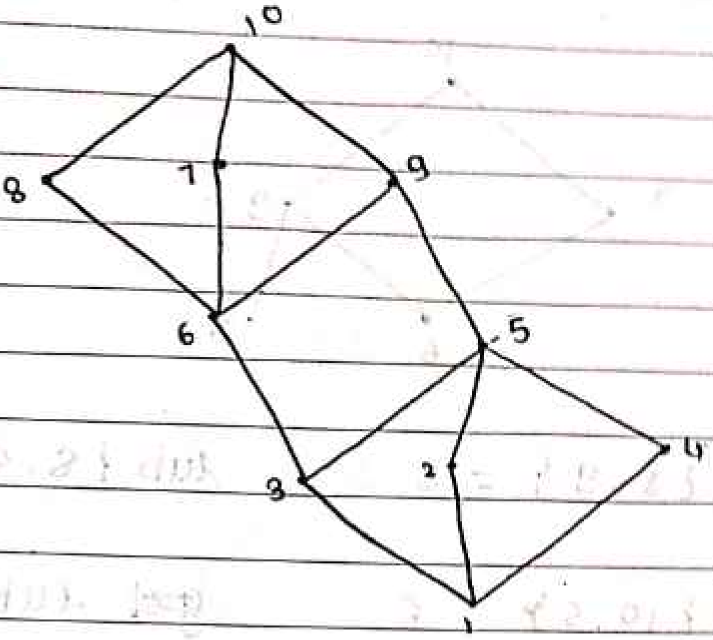
(lub) least upper bound {a, f} → b

Lower bound {a, f} = ∅
greatest lower bound
(glb) → ∅

ub → {c, e} = d
lub {c, e} = d

lb - {c, e} = b, a, f
glb - {c, e} =

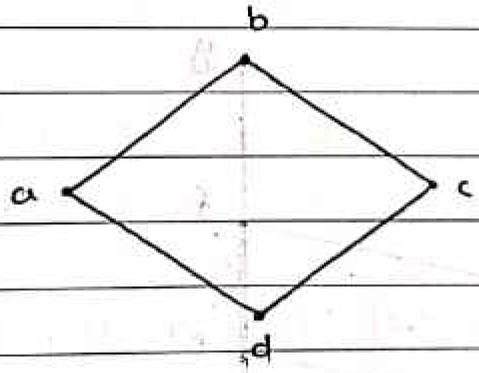
ub - {b, d} =
lub - {b, d} =



glb {2, 8} = 1 lub {3, 2} = 5

glb {2, 7} = 1 lub {4, 8} = 10

glb {5, 8} = 3 lub {3, 5} = 5

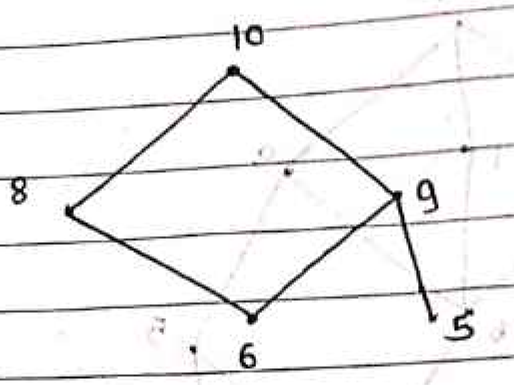


glb {a, d} = d lub {a, d} = b

glb {a, b} = d lub {a, b} = b

glb {a, c} = d lub {a, c} = b

glb {b, d} = ~~b~~d lub {b, d} = b

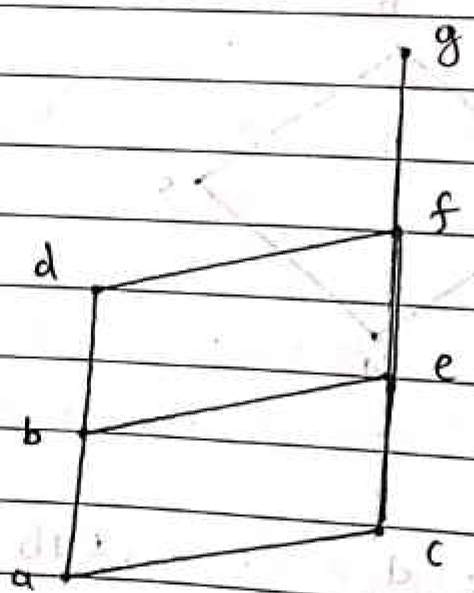


→ glb {8, 9} = 6   lub {8, 9} = 10

glb {10, 6} = 6   ~~gl~~ lub {10, 6} = 10

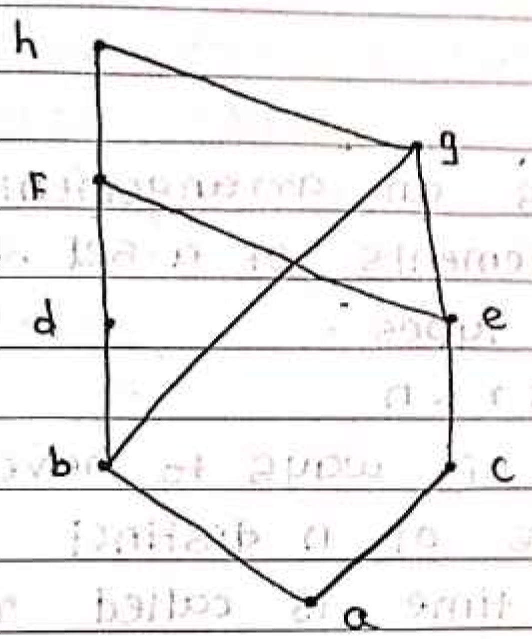
glb {6, 5} = ∅

∴ This hasse diagram is not lattice.



glb {d, e} = b
lub {d, e} = F

glb {b, c} = a
lub {b, c} = e



glb {b, c} = a
lub {b, c} = g

glb {f, g} = e, b
lub {f, g} = h

glb {b, e} = a
lub {b, e} = f

∴ This is not a lattice.

- Permutation is an arrangement of sequence of elements of a set.

It has three types.

Type - ① ~~tate~~ let, $0 \le r \le n$

the number of ways to have an ordered sequence of n distinct element taken r at a time is called r permutation of r elements

$$P(n, r) \text{ or } {}^nP_r = \frac{n!}{(n-r)!}$$

$S = \{a, b, c\}$

2 permutation

ab, ac, bc,

ba, ca, cb,

$$P(3,2) = \frac{3!}{(3-2)!}$$

$$= \frac{3 \times 2 \times 1}{1}$$

$$= 6$$

4 person enter a bus in which there are 6 vacant seats in how many whys can they take their place.

→ $P(6,4)$

$n = 6$

$r = 4$

$$P(6,4) \text{ or } {}^{6}P_{4} = \frac{6!}{(6-4)!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times \cancel{2} \times \cancel{1}}{\cancel{2} \times \cancel{1}}$$

$$= 30 \times 12$$

$$= 360$$

Q. Find a permutation of a set $A = \{1,2,3,4$ Taking the elements two at a time

→ $n = 4$

$r = 2$

$$P(4,2) = \frac{4!}{(4-2)!}$$

$$= \frac{4 \times 3 \times 2 \times 1}{2 \times 1}$$

$$= 12$$

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Q. Find how many words of length 3 can be form from the word COMPUTER. The beginning letter must be c & there should be no repeatation.

→ n = 7
r = 2

$$P(7,2) = \frac{7!}{(7-2)!}$$

$$= \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$$

$$= 42$$

Type - ② The no. of ways in which of the n element can be arranged where r_1 element are of the one kind r_2 element are of another kind and so on till r_k elements of another kind is Given by Formula.

$$\frac{P(n,r)}{r_1! \cdot r_2! \cdots r_k!}$$

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Q. In How many ways the letters in a word MISSISSIPPI can be arranged.

→ MISSISSIPPI

$P(n,r) = 11!$

$$\frac{P(n,r)}{r_1!\, r_2!\, r_3!} = \frac{11!}{4!\, 4!\, 2!}$$

$$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!} \cdot 4! \times 2!}$$

$$= \frac{11 \times \overset{5}{\cancel{10}} \times \overset{3}{\cancel{9}} \times \overset{2}{\cancel{8}} \times 7 \times \overset{3}{\cancel{6}} \times 5}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{2} \times 1}$$

$$= 11 \times 5 \times 3 \times 2 \times 7 \times 3 \times 5$$

$$= 34650$$

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Type ③

The Number of permutation of an element are at a time smaller, when each element may be replaced once twice are time in any arrangement the number of way in which replaces can be Filled up by

Q. A die is rolled 3 times find the No. of faces that appears on top

→ $6 \times 6 \times 6$

$r = 3$

$n = 6$

$n^r = 6^3$

Q Find the No. of binary sequence of length 5

→ 1 0

— — — — —

$r = 5$

$n = 2$

$n^r = 2^5$

Combination .

- Let $0 \geq r \geq n$

A selection of set r element are from a set of n distinct element is called combinations .

$$C(n,r) = \frac{n!}{r!\,(n-r)!}$$

e.g.

$r = 2$

$n = 3$

$$C(n,r) = \frac{3!}{2!\,(3-2)!}$$

$$= \frac{3\times 2\times 1}{2\times 1\times (1)!}$$

$C(3,2) = 3$

Q. In how many ways can 25 let admitted students be assign to three practicle batches if the first batch can accomoded 10 students the second 8 and the third by 7

→ $n = 25$

$C(25,10)$

$C(15,8)$

$C(7,7)$

$C(25,10) \times C(15,8) \times C(7,7)$

$$= \left[\frac{25!}{10!\,(25-10)!}\right] \times \left[\frac{15!}{8!\,(15-8)!}\right] \times \left[\frac{7!}{7!\,(7-7)!}\right]$$

$$= \frac{25!}{10!\,15!} \times \frac{15!}{8!\,7!} \times \frac{7!}{7!\,(0)!}$$

Q A and B are the member of club the club has membership of 30. In how many ways can a commitee of 10 be formed.

i) A must be included in a committee.

ii) A or B should included but not both

→

i) $\cancel{29!}$

$C(29, 9)$

$$= \frac{29!}{9!\,(29-9)!}$$

$$= \frac{29!}{9! \times 20!}$$

$$= \frac{29 \times \overset{7}{\cancel{28}} \times \overset{3}{\cancel{27}} \times 26 \times \overset{5}{\cancel{25}} \times \overset{3}{\cancel{24}} \times 23 \times \overset{11}{\cancel{22}} \times \overset{3}{\cancel{21}} \times \cancel{20!}}{\underset{1}{\cancel{9}} \times \underset{1}{\cancel{8}} \times \underset{1}{\cancel{7}} \times 6 \times \underset{1}{\cancel{5}} \times 3 \times \underset{1}{\cancel{4}} \times \underset{1}{\cancel{2}} \times 1 \times \cancel{20!}}$$

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$$= \frac{29 \times 7 \times 3 \times 26 \times 5 \times 3 \times 23 \times 11 \times 8}{8}$$

$= 203 \times 178 \times 15 \times 253$

$= 51359 \times 1170$

$= 60090030$

ii)

- A is included
  $C(28, 9)$

- B is included
  $C(28, 9)$

$= C(28, 9) + C(28, 9)$

$=$

Q A die is rolled 6 times & a sequence of faces is noted in How many sequences does the face 5 appear and even numbers of time also find the number of sequences in which 5 appears exactly twice or the face 3 appear exactly 4 times.

→

(1) 1) 0 times

$C(6,0) \times 5^6$

2) 2 times

$C(6,2) \times 5^4$

3) 4 times

$C(6,4) \times 5^2$

4) 6 times

$C(6,6) \times 5^0$

$[C(6,0) \times 5^6] + [C(6,2) \times 5^4] + [C(6,4) \times 5^2] \times [C(6,4) \times 5^2]$

$= \left[\frac{6!}{0!(6-0)!} \times 5^6\right] + \left[\frac{6!}{2!(6-2)!} \times 5^4\right] + \left[\frac{6!}{4!(6-4)!} \times 5^2\right] + \left[\frac{6!}{6!(6-6)!} \times 5^0\right]$

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$$= \left[\frac{6!}{1 \cdot 6!} \times 5^6\right] + \left[\frac{6!}{2! \times 4!} \times 5^4\right] + \left[\frac{6!}{4! \times 2!} \times 5^2\right] + \left[\frac{6!}{6!\ 0!} \times 5^0\right]$$

$$= \left[1 \times 5^6\right] + \left[\frac{6 \times 5 \times 4!}{2 \times 1 \times 4!} \times 5^4\right] + \left[\frac{6 \times 5 \times 4!}{4! \times 2 \times 1} \times 5^2\right] + \left[1 \times 1\right]$$

$$= [15625] + [15 \times 625] + [15 \times 25] + 1$$

$$= 15625 + 9375 + 375 + 1$$

$$= 25376$$

② $C(6,2) \times 5^4 + C(6,4) \times 5^2 - C(6,2)$

$$= \frac{6!}{2!\,(6-2)!} \times 5^4 + \frac{6!}{4!\,(6-4)!} \times 5^2 - \frac{6!}{2!\,(6-2)!}$$

$$= \frac{6 \times 5 \times 4!}{2 \times 1 \times 4!} \times 5^4 + \frac{6 \times 5 \times \cancel{4!}}{\cancel{4!} \times 2!} \times 5^2 - \frac{6 \times 5 \times 4!}{2 \times 1 \times 4!}$$

$$= [15 \times 5^4] + [15 \times 5^2] - [15]$$

$$= [9375] + [375] - 15$$

$$= 9750 - 15 = 9735$$

Discrete Numeric function.

Q. A ping pong ball is droped to the floor from a height of 20 m. suppose at the ball always rebounds to reach half of the height which it falls then find

① determine the Numeric function A where A are is height of the ball reaches in earth rebound.

→ $a_0 = 20$

$a_1 = 10$

$a_2 = 5$

$a_3 = 2.5$

$a_4 = 1.25$

⋮

$a = \{a_0, a_1, a_2, a_3, a_4, \ldots\}$

$a = \{20, 10, 5, 2.5, 1.25, \ldots\}$

$$a_r = \begin{cases} 20 & , r = 0 \\ \frac{a_{r-1}}{2} & , r > 0 \end{cases}$$

② If we are denotes the loss of in height during the earth rebound

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then express b_r in terms of a_r

$b_r = a_0 - a_r$

$b_0 = 0$

$b_1 = 10$

$b_2 = 15$

$b_3 = 17.5$

$b_4 = 18.75$

⋮

Ramesh deposit 200 Rs in saving account and the interest rate of 9% per year compounded annualy If a_r denotes the amount in account after r years determine the Numeric Function way.

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$= 200\left(1 + \frac{9}{100}\right)^r$$

$$= 200\,(1 + 0.09)^r$$

$$a_r = 200\,(1.09)^r$$

* Generating Function

$z = \{ z^0 , z^1 , z^2 , z^3 \}$

$a : \{ a_0 , a_1 , a_2 , a_3 , \ldots \}$

$A(z) = \{ a_0 z^0 + a_1 z^1 + a_2 z^2 + a_3 z^3 + \ldots \}$

Numeric Function		A(z)
① a_r = constant e.g. $a = (1, 1, 1, \ldots)$	$\longrightarrow$	$\frac{\text{constant}}{(1-z)}$
② $a_r = r$ e.g. $a = (0, 1, 2, 3)$	$\longrightarrow$	$\frac{z}{(1-z)^2}$
③ $a_r = b_r \cdot a_r$	$\longrightarrow$	$\frac{abz}{(1-az)^2}$
④ $a_r = k \cdot a^r$	$\longrightarrow$	$\frac{k}{1-az}$
⑤ $a_r = \frac{1}{r!}$	$\longrightarrow$	e^z

$a_r = 3^r + 4^{r+1}$

$A(z) = ?$

$b_r = 1 \cdot 3^r$

$c_r = 4^{r+1}$

$$B(z) = \frac{1}{1-3z}$$

$$C(z) = 4^r \cdot 4^1$$

$$= 4 \cdot 4^r$$

$$= \frac{4}{1-4z}$$

$$A(z) = B(z) + C(z)$$

$$= \frac{1}{(1-3z)} + \frac{4}{(1-4z)}$$

$$= \frac{1(1-4z) + 4(1-3z)}{(1-3z)(1-4z)}$$

$$= \frac{1-4z+4-12z}{(1-3z)(1-4z)}$$

$$= \frac{5-16z}{(1-3z)(1-4z)}$$

Recurrence Relation

- A recurrence relation of the sequence $\{a_n\}$ is an equation on that expresses a_n in the terms of One or more of the previous term of the sequence namely $a_0, a_1, a_2, \ldots a_{n-1}$ for all integers $n \geq n_0$ where n_0 is non negative integer.

$$C_0 a_r + C_1 a_{r-1} + C_2 a_{r-2} + \ldots + C_k a_{r-k} = f(r)$$

$$\sum_{i=0}^{k} C_i a_{r-i} = f(r)$$

$C_0 \neq 0$ & $C_k \neq 0$ ... k^{th} order

2nd - order

- $C_0 a_r + C_1 a_{r-1} + C_2 a_{r-2}$

3rd order

- $C_0 a_r + C_2 a_{r-1} + C_2 a_{r-2} + C_3 a_{r-3}$

Total

$$a_r = a_r^{(h)} + a_r^{(P)}$$

Homogeneous

$F(r) = 0$

Perticular

$F(r) \neq 0$

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$$C_0 a_r + C_1 a_{r-1} + C_2 a_{r-2} = 0$$

Auxillary eqn.

$$C_0 m^2 + C_1 m + C_2 = 0$$

CASE I : Roots are real & equal

$$m_1 = 2$$
$$m_2 = 2$$
$$m_1 = m_2$$

General soln $= a_r = (C_1 + rC_2) m^r$

CASE II : Roots are real & unequal

$$m_1 = 2$$
$$m_2 = 3$$
$$m_1 \neq m_2$$

General soln =

$$a_r = C_1 m^r + C_2 m^r$$

CASE III : Roots are complex / Imaginary

$$m = \alpha \pm i\beta$$

$$a_r = (C_1 \cos r\theta + C_2 \sin r\theta) R^r$$

$$R = \sqrt{\alpha^2 + \beta^2}$$

$$\theta = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$$

Q. $a_r + 5a_{r-1} + 6a_{r-2} = 0$

→ A.E

$m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

$(m+2)(m+3) = 0$

$m = -2, -3$

G.S

$a_r = c_1 m^r + c_2 m^r$

$= c_1(-2)^r + c_2(-3)^r$

Q $a_r - 10a_{r-1} + 9a_{r-2} = 0$

→ Given :

$a_0 = 3$

$a_1 = 11$

A.E

$m^2 - 10m + 9 = 0$

$m^2 - 10m + 9 = 0$

$m^2 - 9m - m + 9 = 0$

$m(m-9) - 1(m-9) = 0$

$(m-9)(m-1) = 0$

$m = 9, 1$

G.S.

$a_r = c_1 m^r + c_2 m^r$

$a_r = c_1 (9)^r + c_2 (1)^r$ —— (*)

$a_0 = c_1(9)^0 + c_2(1)^0$

$3 = c_1 + c_2$

$c_1 + c_2 = 3$ — ①

$c_1 = 3 - c_2$ — ②

$a_1 = c_1(9)^1 + c_2(1)^1$

$11 = 9c_1 + 1c_2$

$9c_1 + c_2 = 11$ — ③

Put eqn ② in ③

$9(3 - c_2) + c_2 = 11$

$27 - 9c_2 + c_2 = 11$

$-9c_2 + c_2 = 11 - 27$

$-8c_2 = -16$

$8c_2 = 16$

$c_2 = \frac{16}{8}$

$\boxed{c_2 = 2}$

Put value of c_2 in eqn ①

$c_1 + c_2 = 3$

$c_1 + 2 = 3$

$\boxed{c_1 = 1}$

Put values of c_1 & c_2 in ⊛

$a_r = 9^r + 2(1)^r = 9^r + 2$

$a_r - 8a_{r-1} + 16a_{r-2} = 0$

$a_2 = 16$

$a_3 = 8$

$\rightarrow$

A . E

$m^2 - 8m + 16 = 0$ 16 8 -4 -4

$m^2 - 4m - 4m + 16 = 0$

$m(m-4) - 4(m-4) = 0$

$(m-4)(m-4) = 0$

$m = 4, 4$

G . S

$a_r = c_1(m)^r + c_2(m)^r$

$a_r = c_1(4)^r + c_2(4)^r$

$a_2 = c_1(4)^2 + c_2(4)^2$

$16 =$

G . S .

$a_r = (c_1 + c_2 r)\, m^r$

$a_r = (c_1 + c_2 r)(4)^r$

$a_2 = (c_1 + c_2(2))(4)^2$

$16 = (c_1 + 2c_2)\, 16$

$16 = 16c_1 + 32c_2$

$1 = c_1 + 2c_2$ —— (1)

$c_1 = 1 - 2c_2$ —— (2)

$$a_3 = (C_1 + C_2(3))(4)^3$$

$$8 = (C_1 + 3C_2)(64)$$

$$8 = 64C_1 + 192C_2$$

$$1 = 8C_1 + 24C_2$$

$$8C_1 + 24C_2 = 1 \quad \text{— ③}$$

Put eqn ② in ③

$$8(1 - 2C_2) + 24C_2 = 1$$

$$8 - 16C_2 + 24C_2 = 1$$

$$8 - 8C_2 = 1$$

$$8 - 1 = 8C_2$$

$$-7 = 8C_2$$

$$\boxed{C_2 = \frac{-7}{8}}$$

Put $C_2 = \frac{7}{8}$ in eqn ①

$$C_1 + 2C_2 = 1$$

$$C_1 + 2\left(\frac{-7}{8}\right) = 1$$

$$C_1 + \left(\frac{-7}{4}\right) = 1$$

$$C_1 = 1 - \left(\frac{-7}{4}\right)$$

$$C_1 = 1 + \frac{7}{4}$$

$$\boxed{C_1 = \frac{11}{4}}$$

$$a_r = (C_1 + C_2 r)(m)^r$$

$$= \left(\frac{11}{4} + \left(\frac{-7}{8}\right) r\right)(4)^r$$

$$= \left(\frac{11}{4} - \frac{7}{8} r\right)(4)^r$$

Q. $a_r - 3a_{r-1} + 3a_{r-2} - a_{r-3} = 0$

$\longrightarrow$ $a_0 = 1$, $a_1 = -2$, $a_2 = -2$

A.E

$$m^3 - 3m^2 + 3m - 1 = 0$$

$$m = 1, 1, 1$$

G.S.

$a_r = (C_1 + C_2 r)(m)^r$ — for 2nd order

$= (C_1 + C_2 r + r^2 C_3)(m)^r$ — for 3rd order

$$a_r = (C_1 + C_2 r + r^2 C_3)(1)^r$$

$$a_0 = (C_1 + C_2(0) + (0)^2 C_3)(1)^0$$

$$1 = (C_1 + 0 + 0)(1)$$

$$1 = C_1$$

$$\boxed{C_1 = 1}$$

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$a_1 = (C_1 + (1) C_2 + (1)^2 (3)) (1)^1$

$-2 = (C_1 + C_2 + C_3) (1)$

$-2 = (1 + C_2 + C_3) (1)$

$-2 - 1 = C_2 + C_3$

$-3 = C_2 + C_3$ —— (1)

$C_2 = -3 - C_3$ —— (*)

$a_2 = (C_1 + (2) C_2 + (2)^2 C_3) (1)^2$

$-2 = (C_1 + 2C_2 + 4C_3) (1)$

$-2 = 1 + 2C_2 + 4C_3$

$-3 = 2C_2 + 4C_3$ —— (2)

$-3 = 2(-3 - C_3) + 4C_3$

$-3 = -6 - 2C_3 + 4C_3$

$-3 + 6 = 2C_3$

$3 = 2C_3$

$$\boxed{C_3 = \frac{3}{2}}$$

put $C_3 = \frac{3}{2}$ in eq^n 1

$-3 = C_2 + C_3$

$-3 = C_2 + \frac{3}{2}$

$-3 - \frac{3}{2} = C_2$

$\frac{-9}{2} = C_2$

$$\boxed{C_2 = \frac{-9}{2}}$$

$a_r = (C_1 + rC_2 + r^2 C_3)(m)^r$

$= \left(1 + r\left(-\frac{9}{2}\right) + r^2\left(\frac{3}{2}\right)\right)(1)^r$

$= \left(1 - \frac{9}{2}r + \frac{3}{2}r^2\right)(1)^r$

$= 1 - \frac{9}{2}r + \frac{3}{2}r^2$

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$$a_r + 2a_{r-1} + 2a_{r-2} = 0$$

$$a_0 = 0 \quad , \quad a_1 = -1$$

→ A.E

$$m^2 + 2m + 2 = 0$$

$$m = -1 \pm i$$

G.S.

$$a_r = (C_1 \cos r\theta + C_2 \sin r\theta) R^r$$

$$R = \sqrt{\alpha^2 + \beta^2}$$

$$R = \sqrt{(-1)^2 + (1)^2}$$

$$= \sqrt{1+1}$$

$$R = \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$$

$$= \tan^{-1}\left(\frac{1}{-1}\right)$$

$$\theta = \tan^{-1}(-1)$$

$$= \pi - \frac{\pi}{4}$$

$$\theta = \frac{4\pi - \pi}{4} = \frac{3\pi}{4}$$

$$a_r = (c_1 \cos r \frac{3\pi}{4} + c_2 \sin r \frac{3\pi}{4})(\sqrt{2})^r$$

$$a_0 = (c_1 \cos(0) \frac{3\pi}{4} + c_2 \sin(0) \frac{3\pi}{4})(\sqrt{2})^0$$

$$= (c_1 + 0)(1)$$

$$0 = c_1$$

$$\boxed{c_1 = 0}$$

$$a_1 = (c_1 \cos(1) \frac{3\pi}{4} + c_2 \sin(1) \frac{3\pi}{4})(\sqrt{2})^1$$

$$= (c_1 \cos \frac{3\pi}{4} + c_2 \sin \frac{3\pi}{4})(\sqrt{2})$$

$$-1 = (0(\cos \frac{3\pi}{4}) + c_2 \sin \frac{3\pi}{4})\sqrt{2}$$

$$\frac{-1}{\sqrt{2}} = c_2 \sin \frac{3\pi}{4}$$

$$\frac{\cancel{-1}}{\cancel{\sqrt{2}}} = \frac{\cancel{\sqrt{2}}}{\cancel{2}} \cancel{c_2}$$

$$\cancel{c_2} = \cancel{-2}$$

$$\frac{-1}{\sqrt{2}} = \frac{\sqrt{2}}{2} c_2$$

$$\frac{-2}{\sqrt{2}\sqrt{2}} = c_2$$

$$\boxed{c_2 = -1}$$

$$a_r = \left(0 \cdot \cos r \frac{3\pi}{4} + (-1) \sin r \frac{3\pi}{4}\right) (\sqrt{2})^r$$

$$a_r = \left(0 - 1 \sin \frac{r\, 3\pi}{4}\right) (\sqrt{2})^r$$

$$a_r = \left(-1 \sin r \frac{3\pi}{4}\right) (\sqrt{2})^r$$

for Perticular soln -

$$C_0 a_r + C_1 a_{r-1} + C_2 a_{r-2} \dots C_k a_{r-k} = f(r)$$

$f(r) \neq 0$

$$a_r = a_r^{(h)} + a_r^{(p)}$$

$$a_r^{(p)}$$

case : $f(r) = b^r$ e.g. 5^r

$a^r = A \cdot b^r$ ..... where b is the root .

Example .

* $a_r - 7a_r + 10 a_{r-2} = 3^r$ —— *

$a_0 = 0$

$a_1 = 1$

→ A.E

$$m^2 - 7m + 10 = 0$$

$$(m-5)(m-2) = 0$$

$$m = 2, 5$$

Homogeneous

$$a_r^{(h)} = c_1 m^r + c_2 m^r$$
$$= c_1(5)^r + c_2(2)^r$$

Particular

$$a_r^{(P)} = A b^r$$
$$a_r^{(P)} = A \cdot 3^r \quad — ①$$

Put ① in eqn *

$$A \cdot 3^r - 7(A 3^{r-1}) + 10(A 3^{r-2}) = 3^r$$

$$A 3^r - \frac{7A \cdot 3^r}{3} + \frac{10 A 3^r}{3^2} = 3^r$$

$$A \cdot 3^r \left[1 - \frac{7}{3} + \frac{10}{9}\right] = A 3^r$$

$$A\left[-\frac{4}{3} + \frac{10}{9}\right] = 1$$

$$A\left[\frac{-12}{9} + \frac{10}{9}\right] = 1$$

$$A\left[\frac{-2}{9}\right] = 1$$

$$A = \frac{-9}{2}$$

$$a_r^{(p)} = \frac{-9}{2} 3^r$$

Total a_r

$$a_r = a_r^{(p)} + a_r^{(h)}$$

$$= -\frac{9}{2} 3^r + C_1 (2)^r + C_2 (5)^r$$

$$= -\frac{3^2 \cdot 3^r}{2} + C_1 (2)^r + C_2 (5)^r$$

$$= -\frac{3^{r+2}}{2} + C_1 (2)^r + C_2 (5)^r \quad \text{—— \#}$$

$r = 0$

$$a_0 = -\frac{3^{0+2}}{2} + C_1 (2)^0 + C_2 (5)^0$$

$$0 = \frac{-9}{2} + C_1 + C_2$$

$$C_1 + C_2 = \frac{9}{2} \quad \text{—— (2)}$$

$r = 1$

$$a_1 = -\frac{3^{1+2}}{2} + C_1 (2)^1 + C_2 (5)^1$$

$$1 = -\frac{27}{2} + 2C_1 + 5C_2$$

$$2C_1 + 5C_2 = \frac{29}{2} \quad \text{—— (3)}$$

from eqn ②

$$C_1 = \frac{9}{2} - C_2$$

eqn ③

$$2\left(\frac{9}{2} - C_2\right) + 5C_2 = \frac{29}{2}$$

$$9 - 2C_2 + 5C_2 = \frac{29}{2}$$

$$9 + 3C_2 = \frac{29}{2}$$

$$3C_2 = \frac{29}{2} - 9$$

$$3C_2 = \frac{29 - 18}{2}$$

$$3C_2 = \frac{11}{2 \cancel{\times 3}}$$

$$C_2 = \frac{11}{2 \times 3}$$

$$\boxed{C_2 = \frac{11}{6}}$$

Put $C_2 = \frac{11}{6}$ in eqn ②

$$C_1 + \frac{11}{6} = \frac{9}{2}$$

$a_r^{(p)} = A \cdot r \cdot b^r$

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$c_1 = \frac{9}{2} - \frac{11}{6}$

$\boxed{c_1 = \frac{16}{6}}$

$\therefore$ Put values of c_1 & c_2 in #

$$a_r = \frac{-3^{r+2}}{2} + \frac{16}{6}(2)^r + \frac{11}{6}(5)^r$$

* $a_r - 3a_{r-1} + 2a_{r-2} = 2^r$ — *

$\rightarrow$

A.E.

$m^2 - 3m + 2 = 0$

$(m-2)(m-1) = 0$

$m = 1, 2$

Homogeneous:

$a_r^{(h)} = c_1 m^r + c_2 m^r$

$a_r^{(h)} = c_1 (1)^r + c_2 (2)^r$

Particular

$a_r^{(p)} = A r b^r$

$a_r^{(p)} = A r 2^r$ —— ①

Put $a_r^{(p)}$ in eqn *

$$A\cdot r\cdot 2^r - 3\left[A(r-1)\, 2^{r-1}\right] + 2\left[A(r-2)\, 2^{r-2}\right] = 2^r$$

$$Ar\cdot 2^r - 3\left[(Ar - A)\, 2^{r-1}\right] + 2\left[(Ar - 2A)\, 2^{r-2}\right] = 2^r$$

$$Ar\cdot 2^r - 3\left[\frac{Ar2^r}{2} - \frac{A2^r}{2}\right] + 2\left[\frac{Ar\cdot 2^r}{2^2} - \frac{2A2^r}{2^2}\right] = 2^r$$

$$A\cdot r\cdot 2^r - \frac{3\cdot A\cdot r\cdot 2^r}{2} + \frac{3\cdot A\cdot 2^r}{2} + \frac{2\cdot A\cdot r\cdot 2^r}{2^2} - \frac{2\cdot 2\cdot A\cdot 2^r}{2^2} = 2^r$$

$$\frac{2\cdot A\cdot r\cdot 2^r - 3\cdot A\cdot r\cdot 2^r}{2} + \frac{3A\cdot 2^r}{2} + \frac{A\cdot r\cdot 2^r}{2} - A\cdot 2^r = 2^r$$

$$\cancel{\frac{-1}{2}A\cdot r\cdot 2^r} + \frac{3}{2}A\cdot 2^r + \cancel{\frac{1}{2}A\, r\cdot 2^r} - A\cdot 2^r = 2^r$$

$$\frac{3}{2}A\cdot 2^r - A\cdot 2^r = 2^r$$

$$A\cdot 2^r\left[\frac{3}{2} - 1\right] = 2^r$$

$$A\left[\frac{1}{2}\right] = 1$$

$$\boxed{A = 2}$$

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Put $A = 2$ in eq^n ①

$a_r^{(P)} = A r 2^r$

$= 2 r 2^r$

$a_r = a_r^{(P)} + a_r^{(h)}$

$a_r = 2 \cdot r \cdot 2^r + c_1 (1)^r + c_2 (2)^r$

* $a_r - 5a_{r-1} + 6a_{r-2} = 5^r$ — *

A.E

$m^2 - 5m + 6 = 0$

$(m-2)(m-3) = 0$

$m = 2, 3$

Homogeneous

$a_r^{(h)} = c_1 (m)^r + c_2 (m)^r$

$= c_1 (2)^r + c_2 (3)^r$ — ①

Particular

$a_1^{(P)} = A \cdot b^r$

$= A \cdot 5^r$ — ②

Put eqn ② in eqn *

$$A \cdot 5^r - 5(A5^{r-1}) + 6(A5^{r-2}) = 5^r$$

$$A5^r - \frac{5A5^r}{5} + \frac{6A5^r}{25} = 5^r$$

$$A5^r \left[1 - \frac{5}{5} + \frac{6}{25}\right] = 5^r$$

$$A\left[1 - 1 + \frac{6}{25}\right] = 1$$

$$A\left[\frac{6}{25}\right] = 1$$

$$A = \frac{25}{6}$$

$$a_r = a_r^{(p)} + a_r^{(h)}$$

$$= A \cdot b^r + C_1 m^r + C_2 m^r$$

$$= \frac{25}{6} \cdot 5^r + C_1 (2)^r + C_2 (3)^r$$

$$= \frac{5^2}{6} \cdot 5^r + C_1 (2)^r + C_2 (3)^r$$

$$= \frac{5^{r+2}}{6} + C_1 (2)^r + C_2 (3)^r$$

$a_r^{(h)}$ for const = A

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* $a_r - a_{r-1} - 6a_{r-2} = -30$

→

A.E

$m^2 - m - 6 = 0$

$(m-3)(m+2) = 0$

$m = 3, -2$

Homogeneous

$a_r^{(h)} = C_1 m^r + C_2 m^r$

$= C_1 (3)^r + C_2 (-2)^r$

Particular

$a_r^{(P)} = A$

$A - A - 6A = -30$

$-6A = -30$

$A = \frac{30}{6}$

$A = 5$

$a_r = a_r^{(P)} + a_r^{(h)}$

$= 5 + C_1 (3)^r + C_2 (-2)^r$

$r = 0$

$a_0 = 5 + C_1 (3)^0 + C_2 (-2)^0$

$20 = 5 + C_1 + C_2$

$C_1 + C_2 = 15$

$r = 1$

$a_1 = 5 + C_1 (3)^1 + C_2 (-2)^1$

$-5 = 5 + 3C_1 - 2C_2$

$3C_1 + -2C_2 = -10$

$3(15 - C_2) - 2C_2 = -10$

$45 - 3C_2 - 2C_2 = -10$

$-5C_2 = -10 - 45$

$-5C_2 = -55$

$C_2 = \frac{55}{5}$

$\boxed{C_2 = 11}$

$C_1 + C_2 = 15$

$C_1 + 11 = 15$

$\boxed{C_1 = 4}$

$a_r = 5 + 4(3)^r + 11(-2)^r$